Appendix A

Set of algebraic equations $F_i$ that the pair of eigenvelocity vectors $\nu$ and $u$ and its eigenvalue $\beta$ satisfy

Consider two vectors $\nu$ and $u$ that relate to each other in general as

$$b\nu^i = a_k^i u^k$$

(A1)

$$b'\nu^i = a_k^i v^k$$

(A2)

where $a$ and $a'$ are tensors of type $(1, 1)$ and $b$ and $b'$ are two scalar variables. Rewriting (A1) and (A2) with $\nu$ and $u$ in their coordinate form we get

$$bdx^i/ds = a_k^i d'x^k/ds'$$

(A3)

$$b'dx^i/ds' = a_k^i dx^k/ds$$

(A4)

where $x^i$ is the time coordinate and $s$ and $s'$ are path parameters. Let $b$, $b'$ and the two path parameters be adjusted so that

$$b(ds'/ds) = b'(ds/ds') = \beta$$

(A5)

where $\beta$ is a scalar variable. Then (A1) and (A2) become

$$\beta\nu^i = a_k^i u^k$$

(A6)

$$\beta u^i = a_k^i v^k$$

(A7)

where

$$v^i = dx^i/ds$$

(A8)

$$u^i = d'x^i/ds$$

(A9)

The path parameter $s$ is the ET-equivalent of proper time in Relativity.

Equations (A6) and (A7) can be re-written as follows.

$$\beta g_{ij}\nu^i = (g_{ij} + h_{ij})u^j$$

(A10)

$$\beta g'_{ij}u^j = (g'_{ij} + h'_{ij})v^j$$

(A11)

where $g$ and $g'$ are symmetric and $h$ and $h'$ are antisymmetric. The condition that $\nu$ and $u$ are eigenvectors is $a'$ in (A7) is the transpose of $a$ in (A6) (ref.1). This condition is satisfied if

$$g' = g$$

(A12)

$$h' = -h$$

(A13)

Re-writing (A10) and (A11) to include (A12) and (A13) we have

$$\beta g_{ij}\nu^i = (g_{ij} + h_{ij})u^j$$

(A14)

$$\beta g_{ij}u^j = (g_{ij} - h_{ij})\nu^j$$

(A15)

In Minkowski spacetime let the coordinate axes at the point of intersection of $\nu$ and $u$ be so configured that $\nu$ and $u$ are coplanar with the time-axis and a space-axis, as shown in Fig. A1.

In this case, (A14) and (A15) produce the following.

$$\sqrt{1 - s^2}\begin{bmatrix} v^0 \\ v^1 \end{bmatrix} = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u^0 \\ u^1 \end{bmatrix}$$

(A16)

$$\sqrt{1 - s^2}\begin{bmatrix} u^0 \\ u^1 \end{bmatrix} = \begin{bmatrix} 1 & -s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v^0 \\ v^1 \end{bmatrix}$$

(A17)

$$s = h_{01}$$

(A18)

According to these the two eigenvelocity vectors $\nu$ and $u$ are Lorentzian boosts of each other with a boost speed of $s$.

Appendix B

The mutual transport equations of $\nu$ and $u$

Let an arbitrary infinitesimal perturbation $\delta x^i$, $i = 0,...,n-1$, be applied to the point of intersection of the pair of eigenvelocity vectors $\nu$ and $u$ so that $\beta$ is kept invariant. Then according equations (5) in §1.2 we have

$$\delta(g_{ij}\nu^i u^j) = \delta \beta = 0$$

(B1)

$$\delta(h_{ij}\nu^i u^j) = \delta \beta (1 - \beta^2) = 0$$

(B2)

On expanding (B1) we get

$$\left[ \frac{d}{ds} \left( \frac{d'x^i}{ds} \right)^2 + g_{kn} \frac{d'}{ds} \left( \frac{dx^k}{ds} \right) + 2g_{kl}(ij,o) \frac{d'x^i}{ds} \frac{dx^j}{ds} \right] \delta x^k = \beta_g$$

(B3)

where

$$\{ij,k\}^2 = \frac{1}{2} g^{ik} \left( \frac{\partial g_{ij}}{\partial x^l} + \frac{\partial g_{ij}}{\partial x^l} - \frac{\partial g_{ji}}{\partial x^l} \right)$$

(B4)

$$\beta_g = \frac{d}{ds} \left( g_{ij} \delta x^i \frac{d'x^j}{ds} \right) + \frac{d'}{ds} \left( g_{ij} \delta x^i \frac{dx^j}{ds} \right)$$

(B5)
Since $\delta x^i$ is arbitrary, (B3) effectively represents $n$ equations, which respectively correspond to the $n$ elements of $\delta x$.

Let $\delta x'$ be an element of $\delta x$ with the remaining elements of $\delta x$ set to zero. Let the scalars within the curly brackets in (B5) be denoted by $c_i$ and $c^i$ as follows.

$$g_{ij} \delta x^i \frac{d'x^j}{ds} = c^i$$  \hspace{1cm} (B5a)

$$g_{ij} \delta x^i \frac{d'x^j}{ds} = c^i$$  \hspace{1cm} (B5b)

The summation on $l$ is suspended in (B5a) and (B5b). Eliminating $\delta x'$ from (B5a) and (B5b) we get

$$(g_{ij}/c^i) \frac{d'x^j}{ds} = (g_{ij}/c^j) \frac{dx^j}{ds}$$  \hspace{1cm} (B5c)

In the $n$ independent equations that correspond to $l = 0, \ldots, n-1$, which (B5c) represents, the ratios between $c^i$ and $c^j$ can be maintained constant using the set of $n$ arbitrary elements of $g$.

Then because $x'$ is arbitrary, $c^i$ and $c^j$ can be individually maintained constant. Therefore

$$\beta_g = 0$$  \hspace{1cm} (B6)

Then we have

$$\left[ g_{kn} \frac{d}{ds} \left( \frac{d'x^n}{ds} \right) + g_{kn} \frac{d'}{ds} \left( \frac{dx^n}{ds} \right) + 2g_{ko}(ij,o) \frac{d'x^i d'x^j}{ds \ ds} \right] = 0$$  \hspace{1cm} (B7)

On expanding (B2) in the same way that (B1) was expanded, we get

$$\left[ h_{kn} \frac{d}{ds} \left( \frac{d'x^n}{ds} \right) - h_{kn} \frac{d'}{ds} \left( \frac{dx^n}{ds} \right) + 2h_{ko}(ij,o) \frac{d'x^i d'x^j}{ds \ ds} \right] \delta x^k = \beta_h$$  \hspace{1cm} (B8)

where

$$\{ij,k\} = \frac{1}{2} h_{lk} \left( \frac{\partial h_{il}}{\partial x^j} + \frac{\partial h_{jl}}{\partial x^i} + \frac{\partial h_{ij}}{\partial x^l} \right)$$  \hspace{1cm} (B9)

$$\beta_h = \frac{d}{ds} \left( h_{ij} \delta x^i \frac{d'x^j}{ds} \right) + \frac{d'}{ds} \left( h_{ij} \frac{dx^j}{ds} \delta x^i \right)$$  \hspace{1cm} (B10)

The reasoning that was used to obtain (B6) is applicable in this case also as the 1-form that $h$ contains consists of $n$ arbitrary elements. Thus, we get

$$\beta_h = 0$$  \hspace{1cm} (B11)

$$\left[ h_{kn} \frac{d}{ds} \left( \frac{d'x^n}{ds} \right) - h_{kn} \frac{d'}{ds} \left( \frac{dx^n}{ds} \right) + 2h_{ko}(ij,o) \frac{d'x^i d'x^j}{ds \ ds} \right] = 0$$  \hspace{1cm} (B12)

Equations (B7) and (B12) can be re-written as

$$\frac{d}{ds} \left( d'x^k \right) + \frac{d'}{ds} \left( dx^k \right) + 2\{ij,k\} \frac{d'x^i d'x^j}{ds \ ds} = 0$$  \hspace{1cm} (B13)

$$\frac{d}{ds} \left( d'x^k \right) - \frac{d'}{ds} \left( dx^k \right) + 2\{ij,k\} \frac{d'x^i d'x^j}{ds \ ds} = 0$$  \hspace{1cm} (B14)

Adding (B13) and (B14) and subtracting (B14) from (B13) we get the following equations that determine the mutual transport of $v$ and $u$.

$$\frac{d}{ds} \left( d'x^k \right) + \{ij,k\} \frac{d'x^i d'x^j}{ds \ ds} = 0$$  \hspace{1cm} (B15)

$$\frac{d'}{ds} \left( dx^k \right) + \{ij,k\} \frac{dx^i d'x^j}{ds \ ds} = 0$$  \hspace{1cm} (B16)

where

$$\{ij,k\} = \{ij,k\}^a + \{ij,k\} \frac{d'}{ds}$$  \hspace{1cm} (B17)

### Appendix C

#### A simple generalisation of Maxwell’s electromagnetic equations

In ET, the antisymmetric tensor $h$ is the sum of a 2-form and the exterior derivative of a 1-form $p$. This $h$ can be used to generalise the following Maxwell’s electromagnetic equations

$$F_{ij} = \partial_j p_i - \partial_i p_j$$  \hspace{1cm} (C1)

$$\partial_j \left( \sqrt{-g} g^{im} g^{jn} F_{mn} \right) = \sqrt{-g} f^i$$  \hspace{1cm} (C2)

where $J$ is the charge-current density vector. If the 2-form component of $h$, say $h$, is assumed to be related to $J$ as

$$\partial_j \left( \sqrt{-g} g^{im} g^{jn} h_{mn} \right) = -\sqrt{-g} J^i$$  \hspace{1cm} (C3)

then in ET the above Maxwell’s equations become generalised as

$$\partial_j \left( \sqrt{-g} g^{im} g^{jn} h_{mn} \right) = 0$$  \hspace{1cm} (C4)

### Appendix D

#### A simple insight into Planck-Einstein Relation

In 2-dimensional Minkowski spacetime, equation

$$\left( \bar{g}_{ij} \pm \bar{h}_{ij} \right) w^j = 0, \ \det(\bar{g} \pm \bar{h}) = 0$$  \hspace{1cm} (22)

reduces to just the following.

$$\frac{dt}{ds} = \frac{dx}{ds}$$  \hspace{1cm} (D1)

where $dt$ and $dx$ are the time and space increments of the photon travel and $ds$ is path parameter increment yet to be established owing to the null character of $w$. Now $dt/ds$ is the dimensionless photon energy $E$. Therefore, (D1) becomes
\[ E' = \frac{dx}{ds} \] (D2)

Since this energy \( E' \) is dimensionless it has to be multiplied by a fundamental energy constant to obtain \( E \) in the usual units of energy. This constant is Planck energy \((\hbar c/G)^{1/2} c^2\) where \( \hbar \) is Planck’s reduced constant, \( c \) is speed of light and \( G \) is gravitation constant. Then (D2) becomes

\[ E = \left( \frac{\hbar c}{G} \right)^{1/2} c^2 \left( \frac{dx}{ds} \right) \] (D3)

Now (22) is the result of unification of translational and rotational motions of light. This unification in this 2-dimensional case connects \( dx \) with an incremental hyperbolic angle \( d\xi \) using Planck distance as follows.

\[ dx = (\text{Planck distance})d\xi \] (D4)

Since Planck distance is \((\hbar c/e^2)^{1/2}\), (D3) becomes

\[ E = (\hbar c)(d\xi/ds) \] (D5)

If the normal angular velocity is the same as \( d\xi (ds/e) \) then that would define the path parameter \( s \) and (D5) becomes the same as Planck-Einstein relation.

\[ E = h\nu \] (D6)

where \( \nu \) is normal frequency.

### Appendix E

**Solution of (25m), (26m) and (24nm) in §2.4, for static spherically symmetric space**

Equations (25m), (26m) and their auxiliary equations are as follows.

\[ R_{ij} + C_{ij}^s = bg_{ij} \] (25m)

\[ C_{ij}^s = \pm bh_{ij} \] (26m)

\[ \det \left( g \pm \hbar \right) = 0 \] (24nm)

where \( R_{ij} = \{ik,k\}_j^s - \{ij,k\}_k^s - \{mk,k\}_k^s \{ij,m\}_m^s \) (27)

\[ C_{ij}^s = \{ik,k\}_j^s + \{mj,k\}_m^s \{ik,m\}_m^a \] (28)

\[ C_{ij}^a = -(mk,k)_a^{ij} a - \{ij,k\}_k^a \] (29)

\[ \{ij,k\}^s = \frac{1}{2} g^{lk} \left( \partial_l g_{jlt} + \partial_t g_{lj} - \partial_j g_{lt} \right) \] (30)

\[ \{ij,k\}^a = \frac{1}{2} h^{lk} \left( \partial_l h_{jlt} + \partial_t h_{lj} - \partial_j h_{lt} \right) \] (31)

For static spherically symmetric conditions \( g \) and \( \hbar \) in spherical polar coordinates \((t, r, \theta, \phi)\) have the following forms.

\[ g_{ij} = \text{diag}\{e^\nu, -e^\lambda, -r^2, -r^2 \sin^2 \theta\} \] (E1)

\[ h_{ij} = \{ h_{01} = -h_{10} = -e^\alpha, h_{23} = -h_{32} = \sin \theta e^\rho \} \] (E2)

Parameters \( \nu, \lambda, a \) and \( \rho \) are functions of \( r \) only and all other elements of \( \hbar \) are zero. The elements of \( \{ij,k\}^s \) that correspond to \( g \) in (E1) are well established in the literature and they can be found on page 84 of ref. 2. The non-zero elements of \( \{ij,k\}^a \) that correspond to \( \hbar \) in (E2) is as follows.

\[ \{12,2\}^a = -\rho'/2 \] (E3)

\[ \{13,3\}^a = -\rho'/2 \] (E4)

\[ \{21,2\}^a = +\rho'/2 \] (E5)

\[ \{23,0\}^a = +\sin \theta e^\rho - a \rho'/2 \] (E6)

\[ \{31,3\}^a = +\rho'/2 \] (E7)

\[ \{32,0\}^a = -\sin \theta e^\rho - a \rho'/2 \] (E8)

The accent,’ on a symbol denotes differentiation with respect to \( r \). On substituting these elements of \( \{ij,k\}^s \) in the expression for \( C_{ij}^s \) in (29), we get

\[ C_{ij}^s = 0 \] (E9)

Hence, it follows that the parameter \( \hbar \) in (25m) and (26m) is zero. On substituting the above elements of \( \{ij,k\}^a \) in the expression for \( C_{ij}^a \) in (28), we get the following.

\[ C_{00}^s = e^{\nu - \lambda} \nu'(\rho')/2 \] (E10)

\[ C_{11}^s = -(\rho'') + \lambda'(\rho')/2 - (\rho')^2/2 \] (E11)

\[ C_{22}^s = -r e^{-\lambda}(\rho') \] (E12)

\[ C_{33}^s = -r \sin^2 \theta e^{-\lambda}(\rho') \] (E13)

Elements \( \{ij,k\}^s \) of the tensor \( R_{ij} \) in (27) can be found on page 85 of ref. 2 where \( R \) has been denoted as \( G \). On substituting in (25m) these elements \( \{ij,k\}^s \) and the above elements of \( C_{ij}^s \) we get

\[ e^{\nu - \lambda} \left( \frac{1}{2} \nu'' + \frac{1}{4} \lambda' \nu' - \frac{1}{4} \nu^2 - \frac{\nu}{r} \right) + \frac{1}{2} e^{\nu - \lambda} \nu' \rho' = 0 \] (E14)

\[ \frac{1}{2} \nu'' - \frac{1}{4} \lambda' \nu' + \frac{1}{4} \nu^2 - \frac{\lambda'}{r} - (\rho'') \] (E15)

\[ r e^{\nu -} \left( \frac{1}{2} \nu' + \frac{1}{4} \lambda' \nu' - \frac{1}{4} \nu^2 - \frac{\nu}{r} \right) = 0 \] (E16)
The parameters \( \nu, \lambda \) and \( \rho \) that satisfy equations (E14) to (E16) are as follows.

\[
\rho' = \frac{2}{r} \quad \text{(E17)}
\]

\[
e^\nu = -(1 - r/(2M)) \quad \text{(E18)}
\]

\[
e^\lambda = -\frac{1}{(1 - r/(2M))} \quad \text{(E19)}
\]

\( M \) in (E18) and (E19) is a constant of integration. Applying the condition (24nm) to the parameters in (E17) to (E19), \( a \) and \( h \) in (E1) and (E2), become

\[
g_{ij} = \text{diag}\{-\left(1 - \frac{r}{2M}\right), 1/\left(1 - \frac{r}{2M}\right), -r^2, -r^2\sin^2\theta\} \quad \text{(E20)}
\]

\[
h_{ij} = \{ h_{01} = -h_{10} = -1, \quad h_{23} = -h_{32} = kr^2\sin\theta \} \quad \text{(E21)}
\]

where \( k \) is a constant which may be set to unity.

**Appendix F**

**Gravitational redshift due to spacetime curvature of the innermost manifold**

The metric tensor field of the innermost manifold, obtained in Appendix E, for static spherically symmetric space is the following.

\[
g_{ij} = \text{diag}\{-\left(1 - \frac{r}{2M}\right), 1/\left(1 - \frac{r}{2M}\right), -r^2, -r^2\sin^2\theta\} \quad \text{(E20)}
\]

Let a photon be emitted at a distance \( r \) from the origin \( O \) of the system of spherical polar coordinates \( (t, r, \theta, \phi) \), and let its frequency at this point of emission be \( \nu_e \). On reaching \( O \) let the photon frequency become \( \nu_r \). These two frequencies relate to each other as \( \text{(ref.4)} \)

\[
\frac{\nu_e}{\nu_r} = (1 - r/(2M))^{-1/2} \quad \text{(F1)}
\]

This frequency ratio in terms of a recessional speed \( s \), is given by

\[
\frac{\nu_e}{\nu_r} = \sqrt{\frac{1 + s}{1 - s}} \quad \text{(F2)}
\]

Combining (F1) and (F2), we get

\[
s = \frac{r/(2M)}{2 - r/(2M)} \quad \text{(F3)}
\]

For comparing with Hubble’s law, let (F3) be re-written as \( s = Hr \), where

\[
H = \frac{1}{4M - r} \quad \text{(F4)}
\]

According to (F4) the maximum value of \( H \) is \( 1/(2M) \) that occurs at \( r = 2M \). Now \( M \) has been estimated in §6.4 as \( 7.47 \times 10^{10} \) g which works out to \( 1.799 \times 10^{57} \) kpc. Hence the maximum value of \( H \) is \( 8.34 \text{ kms}^{-1}\text{Mpc}^{-1} \). This value of \( H \) is considerably less than the present value of the Hubble’s constant, \( 73.8 \text{ kms}^{-1}\text{Mpc}^{-1} \). Therefore, the gravitational curvature of the innermost manifold has only a small effect on the observed Hubble expansion of the physical universe.

**References**