The gravitational magnetic field is derived from Special Relativity force transformations. In S two identical particles without electric charge (q = 0) and mass m are moving with \( v_{1x} = v_{2x} = u, x_1 = x_2 = x_0, z_1 = z_2 = 0 \) and distance \( r = y_2 - y_1 \) since \( q = 0 \) the only force is the gravitational force \( F'_g \).

\[
F'_g = G \frac{m_0 m_0}{r^2}
\]

In S, gravitational force \( F_g \), \( \gamma^{-1} = (1 - \beta^2)^{1/2}, \beta = v/c, F_g = \gamma^{-1} F'_g \).

The relativistic transformation of net force is [9].

\[
F_B g = (\gamma^{-1} - \gamma^3) F'_g, \quad F_B g = \gamma^3 (\gamma^{-4} - 1)
\]

\[
F'_g, \quad F_B g = \gamma^3 (-2 \beta^2 + \beta^4) F'_g
\]

With \( b = 2v - v^3/c^2, \beta_u = b/c, \gamma_u^{-1} = (1 - \beta_u^2)^{1/2} \), transformations are

\[
B_{gz} = \gamma_g (B'_{gz} + b F'_{gy} / c^2),
\]

\[
B_{gy} = \gamma_g (B'_{gy} - b F'_{gz} / c^2), \quad B_{gx} = B'_{gx}
\]

\[
F_{gz} = \gamma_g (F'_{gz} - b F'_{gy}),
\]

\[
F_{gy} = \gamma_g (F'_{gy} + b F'_{gz}), \quad F_{gx} = F'_{gx}
\]

\[
x' = \gamma_g (x - bt), \quad t' = \gamma_g (t - bx/c^2), \quad y' = y, \quad z' = z
\]

FBg is associated with Dark Matter energy which has been stored in the dark matter energy tensor \( D (4, 0) \) related to the Riemann curvature. Dark matter gravity is generated by \( D (4, 0) \) energy related directly to the \( S (4, 0) \) tensor, however this gravity is attributed to exotic particles never detected, galaxies in our universe are rotating with such speed that the gravity generated by their observable matter could not possibly hold them together.

Total Energy \( T (4, 0) \) and energy tensors are denied to complete the General Relativity field equations. The Ricci decomposition is a way of breaking up the Riemann curvature tensor into three orthogonal tensors, \( Z (4, 0) \), Weyl tensor \( C (4, 0) \) and \( S (4, 0) \).

**Keywords:** Dark Matter, Expansion of the Universe, Riemann Curvature, Ricci Decomposition, Special Relativity, Gravitational Magnetic Field, General Relativity Field Equations
Conformal energy \( U \) defined as a combination of \( C \) and the Hodge dual of \( C \), dark matter energy \( D \) defined as a combination of \( S \) and the Hodge dual of \( S \), similar definitions for \( V/Z \) and \( T/R \).

The Ricci decomposition is a way of breaking up the Riemann curvature tensor into three orthogonal tensors, \( Z \), Weyl tensor \( C \) and \( S \), \( S \) tensor generates the dark matter gravity [6-8].

\[
R_{ijkl} = Z_{ijkl} + C_{ijkl} + S_{ijkl}
\]

\[
S_{ijkl} = \frac{1}{12} R \left( g_{ii}g_{jk} - g_{ik}g_{jl} \right)
\]

\[
Y_{jk} = R_{jk} - \frac{1}{4} R g_{jk}, \quad Z_{ijkl} = \frac{1}{2} \left( Y_{i lj k} - Y_{j il k} - Y_{iklj} + Y_{jikl} g_{ii} \right)
\]

where \( R_{abcd} \) is the Riemann tensor, \( R_{ab} \) is the Ricci tensor, \( R \) is the Ricci scalar (the scalar curvature).

The conformal energy tensor \( U \) can be defined as a combination of \( C \) and the Hodge dual of \( C \) [1-3].

\[
U_{abcd} = \frac{1}{12 \pi} \left( C_{amcd} C_{bnm} + * C_{amcd} * C_{bnm} + C_{amcn} C_{abcd} + * C_{amcn} * C_{abcd} \right)
\]

The new dark matter energy tensor \( D \) can be defined as a combination of \( S \) and the Hodge dual of \( S \).

\[
D_{abcd} = \frac{1}{12 \pi} \left( S_{amcd} S_{bnm} + * S_{amcd} * S_{bnm} + S_{amcn} S_{abcd} + * S_{amcn} * S_{abcd} \right)
\]

The new energy tensor \( V \) can be defined as a combination of \( Z \) and the Hodge dual of \( Z \).

\[
V_{abcd} = \frac{1}{12 \pi} \left( Z_{amcd} Z_{bnm} + * Z_{amcd} * Z_{bnm} + Z_{amcn} Z_{abcd} + * Z_{amcn} * Z_{abcd} \right)
\]

The new Total Energy tensor \( T \) can be defined as a combination of the Riemann tensor \( R \) and the Hodge dual of \( R \).

\[
T_{abcd} = \frac{1}{12 \pi} \left( R_{amcd} R_{bnm} + * R_{amcd} * R_{bnm} + R_{amcn} R_{abcd} + * R_{amcn} * R_{abcd} \right)
\]

**Hodge Dual Definitions**

The Hodge dual definition for Electromagnetic tensor and Weyl tensor [4].

\[
* F_{ab} = \frac{1}{2} \varepsilon_{abln} F^{ln}
\]

\[
* C_{abcd} = \frac{1}{2} \varepsilon_{abcd} C_{ln}^{ln}
\]

The Hodge dual definition for dark matter \( S \) tensor, \( Z \) and \( R \) tensors

\[
* S_{abcd} = \frac{1}{2} \varepsilon_{abcd} S_{ln}^{ln}, \quad * Z_{abcd} = \frac{1}{2} \varepsilon_{abcd} Z_{ln}^{ln}, \quad * R_{abcd} = \frac{1}{2} \varepsilon_{abcd} R_{ln}^{ln}
\]

Weyl tensor \( C \) (4, 0) is related to the new Conformal Energy tensor \( U \) (4, 0). Dark matter tensor \( S \) (4, 0) is related to the new dark matter energy tensor \( D \) (4, 0). \( Z \) (4, 0) tensor is related to the new energy tensor \( V \) (4, 0). Riemann tensor \( R \) (4, 0) is related to the new Total Energy tensor \( T \) (4, 0).

**Complete General Relativity Field Equations**

The complete field equations are described by a new \( T \) (4, 0) tensor for Total Energy, the new conformal energy tensor \( U \) (4, 0), the new energy tensor \( V \) (4, 0) and the new dark matter energy tensor \( D \) (4, 0).

\[
Z_{ab} - \frac{1}{2} Z g_{ab} + \Lambda Z g_{ab} = -k Z V_{ab}
\]

\[
C_{ab} - \frac{1}{2} C g_{ab} + \Lambda C g_{ab} = -k C U_{ab}
\]

\[
S_{ab} - \frac{1}{2} S g_{ab} + \Lambda S g_{ab} = -k S D_{ab}
\]
In the general theory of relativity the Einstein field equations relate the geometry of spacetime to the distribution of matter [5].

\[ R = Z + C + S \]
\[ \kappa T = k_z V + k_c U + k_s D \]
\[ \Lambda = \Lambda_z + \Lambda_c + \Lambda_s \]
\[ \kappa T_{abcd} = k_z V_{abcd} + k_c U_{abcd} + k_s D_{abcd} \]

In the general theory of relativity the Einstein field equations relate the geometry of spacetime to the distribution of matter [5].

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa T_{\mu\nu} \]

Conclusions

When a charged particle is moving it generates a magnetic field that interacts with charged particles in motion, similarly when a particle with mass is moving it generates a gravitational magnetic field that interacts with other particles in motion.

This new force, the gravitational magnetic force has been detected by astronomers for decades but has been misinterpreted, in a galaxy there are no dark matter particles generating the dark matter force, moving stars generate gravitational magnetic fields that interact with other stars in motion.

The gravitational magnetic force is derived from Special Relativity force transformations, if Special Relativity is correct the gravitational magnetic force is irrefutable.

References