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Short Communication

On Generalized Dirac's Equation

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Abstract

In four dimensions, the Minkowski metric $\eta_{\mu\nu}=diag(+1,-1,-1,-1)$ leads to the 16 dimensional Clifford algebra C(1,3), Dirac equation is using four of these 16 matrices that form a basis of this algebra, a new operator is defined using all of these matrices and also generalized for a curved space

Multilevel operator D^{mul}

We are using Pauli matrices σ, electromagnetic four-potential A_μ and charge e with ħ=c=1 [1].

$$I=\begin{pmatrix}1&0\\0&1\end{pmatrix}\quad \sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix}\quad \sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix}\quad \sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$
$$\alpha=\begin{pmatrix}0&\sigma\\\sigma&0\end{pmatrix}\quad \Sigma=\begin{pmatrix}\sigma&0\\0&\sigma\end{pmatrix} \tag{1}$$

In four dimensions, Minkowski's metric $\eta_{\mu\nu}=diag(+1,-1,-1,-1)$ leads to the Clifford algebra C(1,3)[2],$\{\gamma^\mu,\gamma^\nu\}=2\eta^{\mu\nu}\times I_{4\times4}$, Dirac matrices $\gamma^0=\sigma_3\otimes I$, $\gamma^j=i\sigma_2\otimes\sigma_j$, $j=1,2,3$; $\gamma^p=-i\gamma^{14}=-i\gamma^0\gamma^1\gamma^2\gamma^3$

$$\gamma^4=\gamma^0\gamma^1,\gamma^5=\gamma^0\gamma^2,\gamma^6=\gamma^0\gamma^3,\gamma^7=\gamma^1\gamma^2,\gamma^8=\gamma^1\gamma^3,\gamma^9=\gamma^2\gamma^3$$
$$\gamma^{10}=\gamma^0\gamma^1\gamma^2,\gamma^{11}=\gamma^0\gamma^1\gamma^3,\gamma^{12}=\gamma^0\gamma^2\gamma^3,\gamma^{13}=\gamma^1\gamma^2\gamma^3,\gamma^{14}=\gamma^0\gamma^1\gamma^2\gamma^3 \tag{2}$$

Multilevel operator D^n acts on level n, n is the number of matrices in the product of the algebra members, for example, D^3 acts on γ^{10},γ^{11},γ^{12} and γ^{13}. Total multilevel operator $D^{mul} = D^0 + D^1 + D^2 + D^3 + D^4$, the action of D^{mul} on the spinor function vanishes $D^{mul}\Psi = 0$ (3)

$$D^0=-m \tag{4}$$

$$D^1=\gamma^\mu p_\mu - ie\gamma^\mu A_\mu \tag{5}$$

$$D^2=-ie\gamma^\mu\gamma^\nu F_{\mu\nu} \text{ with } F_{\mu\nu}=A_{\mu,\nu}-A_{\nu,\mu} \tag{6}$$

$$D^2=-ie\alpha E+e\Sigma H \tag{7}$$

$$D^3=-ie\gamma^\mu\gamma^\nu\gamma^\delta F_{\mu\nu\delta} \text{ with } F_{\mu\nu\delta}=A_{\mu,\nu}{}_{,\delta}-A_{\mu,\delta}{}_{,\nu}=0 \tag{8}$$

$$D^4=-ie\gamma^\mu\gamma^\nu\gamma^\delta\gamma^\lambda F_{\mu\nu\delta\lambda} \text{ with } F_{\mu\nu\delta\lambda}=A_{\mu,\nu}{}_{,\delta}{}_{,\lambda}-A_{\mu,\nu}{}_{,\lambda}{}_{,\delta}=0 \tag{9}$$

Multilevel operator $D^{mul}(\eta_{\mu\nu}, A_\mu, e)$ can be generalized for a curved space with four-potential P, field charge q and covariant derivative[3] ($;_\mu$) instead of derivative ($,_\mu$) in the definition of p_μ

$$D^{mul}(g_{\mu\nu},P_\mu,q)\Psi=0 \tag{10}$$

$$\{\gamma^\mu,\gamma^\nu\}=2g^{\mu\nu}\times\mathbb{I} \tag{11}$$

$$D^0=-m \tag{12}$$

$$D^1=\gamma^\mu p_\mu - iq\gamma^\mu P_\mu \tag{13}$$

$$D^2=-iq\gamma^\mu\gamma^\nu G_{\mu\nu} \text{ with } G_{\mu\nu}=P_{\mu;\nu}-P_{\nu;\mu} \tag{14}$$

$$G_{\mu\nu}(P)=P_{\mu;\nu}-P_{\nu;\mu}=P_{\mu,\nu}-P_{\nu,\mu}+P_\alpha T^\alpha_{\mu\nu} \tag{15}$$

$$G_{\mu\nu}(P) = F_{\mu\nu}(P) + P_\alpha T^\alpha_{\mu\nu} \quad (16)$$

$$D^2 = -iq\alpha E(P) + q\Sigma H(P) - iq\frac{1}{2}\gamma^\mu\gamma^\nu P_\alpha T^\alpha_{\mu\nu} \quad (17)$$

For gravity $G_{\mu\nu}(P)$ is the new gravitomagnetic tensor. $T^\propto_{\mu\nu}$ is the torsion tensor [4].

$$D^3 = -iq\gamma^\mu\gamma^\nu\gamma^\delta G_{\mu\nu\delta} \text{ with } G_{\mu\nu\delta} = P_{\mu;\nu;\delta} - P_{\mu;\delta;\nu} \quad (18)$$

$G_{\mu\nu\delta}(P) = P_\alpha R^\alpha_{\mu\nu\delta}$ with $R^\alpha_{\mu\nu\delta}$ the Riemann-Christoffel tensor [5]. (19)

$$D^3 = -iq\gamma^0\gamma^1\gamma^2 P_\alpha R^\alpha_{012} - iq\gamma^0\gamma^1\gamma^3 P_\alpha R^\alpha_{013} - iq\gamma^0\gamma^2\gamma^3 P_\alpha R^\alpha_{023} - iq\gamma^1\gamma^2\gamma^3 P_\alpha R^\alpha_{123} \quad (20)$$

$$D^4 = -iq\gamma^\mu\gamma^\nu\gamma^\delta\gamma^\lambda G_{\mu\nu\delta\lambda} \text{ with } G_{\mu\nu\delta\lambda} = P_{\mu;\nu;\delta;\lambda} - P_{\mu;\nu;\lambda;\delta} \quad (21)$$

$$G_{\mu\nu\delta\lambda}(P) = P_{\alpha;\nu} R^\alpha_{\mu\delta\lambda} + P_{\mu;\alpha} R^\alpha_{\nu\delta\lambda} \quad (22)$$

$$D^4 = -iq\gamma^0\gamma^1\gamma^2\gamma^3 P_{\alpha;1} R^\alpha_{023} - iq\gamma^0\gamma^1\gamma^2\gamma^3 P_{0;\alpha} R^\alpha_{123} \quad (23)$$

Conclusions

Multilevel operator $D^{mul}(g_{\mu\nu}, P_\nu, q)$ has been generalized for a curved space with a general four-potential P. For gravity $G_{\mu\nu}(P)$ is the new gravitomagnetic tensor and torsion tensor $T^\propto_{\mu\nu}$ appears in its definition.

In a flat space $G_{\mu\nu}(A) = F_{\mu\nu}(A)$, D^3 and D^4 operators vanish. In a curved space the curvature tensor $R^\propto_{\mu\nu\delta}$ appears in levels 3 and 4.

The appearance of torsion tensor $T^\propto_{\mu\nu}$ and curvature tensor $R^\propto_{\mu\nu\delta}$ in multilevel operator $D^{mul}(g_{\mu\nu}, P_\nu, q)$ means that this operator is a fundamental operator in Quantum Field Theory.

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